POWER SYSTEM RELIABILITY

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by
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B. Gopal

CERTIFICATE

This is to certify that this work on "Power System Reliability" has been carried out under my supervision and it has not been submitted elsewhere for a degree.

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SYNOPSIS

The concept of Markov process is used to derive expressions for an equivalent of two units in series and parallel considering two state fluctuating normal and stormy weathers. These expressions are then used to simplify a system by series and parallel block reduction technique. It is observed that this technique alone is not sufficient for a system where some complex configuration is involved. For such cases Star-Delta conversion in addition with the series and parallel equivalents is used. The results obtained by this technique are compared with those obtained by existing methods. Finally a computer flow chart for system reduction is described.

CONTENTS

CHAPTER - I

INTRODUCTION

Perhaps one of the most challenging problems in this decade is the design and development of large scale systems for commercial purposes. Power system is also one of those large scale commercial systems. In this age and in the ages coming the human race will be more and more dependent upon electricity. This fact introduces the need for reliable supply to consumers. Historically reliability has always been considered during system design. However, as systems have become increasingly complex the reliability problem has become more acute. Much of the early reliability work was confined to making trade offs between certain performance and reliability aspects of electronic components. For example, as the need for high power transmitters developed, it was recognized that increased performance could only be attained at the price of decreased reliability. Obviously increase in power means greater complexity and thus reducing the mean time to failure of the transmitters.

There are various definitions of system reliability depending upon the particular systems in which they are going to be used. However, for power systems one which is widely used is "The degree of assurance needed to deliver electrical power to the load, in other words availability of electrical power".

Basically there are two ways to achieve the reliability. The first is to develop highly reliable parts for use in equipments and systems. The second is to design reliable systems from less reliable parts. The second approach looks to be quite appropriate because it leads to the increase in system reliability as a whole.

The scope of this study is to develop a new technique for the evaluation of reliability for a complex power system. Although reliability aspects are not new their application to power systems has started only recently[1,2]. It is found that the existing methods are either too complicated or less accurate for the evaluation of reliability measures. Therefore this study makes an effort to derive a procedure which is reasonably accurate and can be easily applied to even complex systems.

CHAPTER - II

SYSTEM RELIABILITY CONCEPTS

2.1 Introduction

The reliability function is defined as the probability of the event success for a specified interval of time. It is obtained by summation or integration over that portion of the sample space which defines the occurence of success. This definition of reliability shows that the reliability function is directly related to the probability function. Hence a good knowledge of probability is needed to study the reliability of a system.

There are two quantitative measures of probability. First is a-priori probability or classical probability. Here all the possible outcomes of an experiment are known. For example, when a coin is tossed it is already known that it will be either head or tail and probability of each of these outcomes is 0.5. The second is the experimental probability. Here the predictions about the outcome are based on past experience. For example, if something has happened n times out of N trials, the probability of the success is $\frac{n}{N}$. This prediction becomes better and better as N tends to infinity. It is only this aspect of probability which is used for system reliability studies.

2.2 Need for Study of Power System Reliability

An increasing amount of attention is being focussed on the reliability or continuity of service afforded by transmission,

distribution and generation systems. The increased attention primarily stems from (i) the need to supply improved service as customers become more and more dependent on their electric service and (ii) the desire to use new system voltages and designs whose reliability is not well known to supply heavier loads of the future. Seeing the need of study of reliability one can make reliability models of big power systems and then try to maximize the reliability by putting more redundancy in the system and at the same time keeping the cost a minimum.

2.3 Concept of Power System Reliability

From a system reliability viewpoint the problem of prediction requires the following steps:

(1) Building a mathematical model and defining the reliability measures.

(2) Developing a technique for evaluation of above reliability measures.

(3) Comparing the predicted data and the experimental data.

To do this first,decision rule has to be established and secondly effectiveness of each in terms of some physical attribute has to be expressed. The procedure involves developing mathematical models which simulate the possible outcomes of each alternate design policy. In this way it is possible to compare and then to decide which is the optimum policy.

Measures of effectiveness are generally chosen to represent well defined physical attributes of a system since

they are amenable to mathematical model building, manipulation and measurement. It has been established that the main reliability measures in a maintained system such as power system are:

(1) Availability: The availability of a system is further subdivided into following categories.

   (a) Instantaneous Availability: The probability that the system will be available at time t.

   (b) Average Up Time: The proportion of time in a specified interval (0, T) that the system is available for use.

   (c) Steady State Availability: The probability that the system is available at time $t = \infty$.

(2) Mean Time To Failure (MTTF): The expected time between system failures.

(3) Duration of Single Down Times: Some times it may be preferable to have higher system failure rate with shorter duration of single down time incidents than a lower system failure rate with longer duration of single down time incidents.

One should not think at this moment that these are the only effective measures of power system reliability. In fact the measures are changeable according to requirements. For example, a situation may arise where the cost of system components is very high. In such cases it is meaningful to combine the system performance and above reliability measures into one measure, i.e., the ratio of increase in component availability and increase in component cost above minimum required. After

this the optimization procedure may be to achieve a given level of system reliability at least cost.

Having obtained the reliability model and measures of a system, this study is concerned with the second of the above mentioned steps i.e., developing a suitable technique for reliability evaluation. The third step is rather controversial area and not dealt in this thesis because experimental data of a power system take years for their collection.

2.4 Type of Data Required

The various data required for study of power system reliability can be, in general, categorized into following groups:

(1) Normal and stormy weather component failure rates.*

(2) Normal and stormy weather component repair rates.

(3) Durations of normal and stormy weathers.

After obtaining these data a probability distribution curve may be plotted for each group of data for a component for normal and stormy weathers. It has been found in reference 1

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* For any component the failure rate D is very high in the beginning and in the end of its life. In between this there is a period when rate of failure is quite low and is almost constant. If t_1 is the duration of beginning time and t_2 is the duration of end time when D is very high and T is the intermediate time when D is fairly constant, then for a practical component

$$t_1, t_2 >> T$$

The intermediate time T is only of interest and the component is used only in this period.

that all the above probability distributions are approximately exponential. Therefore if $P_f(t)$ is probability of failure, $P_r(t)$ is the probability of repair and $P_x(t)$ is the probability of a weather duration in time t, then

$$P_f(t) = 1 - e^{-\Gamma t}$$

$$P_r(t) = 1 - e^{-Ut}$$

$$P_x(t) = e^{-t/x}$$

Γ and U in the above expressions are called average or expected failure rate and repair rate of the component. x is called the expected duration of the weather.

2.5 Assumptions

In order to simplify the reliability model of a system the following assumptions are made:

(1) Repair and weather duration distributions are exponential.

(2) The MTTF of the component is exponentially distributed.

(3) Repair rates of the components are much higher than their failure rates.

(4) Only one component fails at a time.

(5) The parallel units in the system are completely redundant.

(6) The system operates in two state fluctuating normal and stormy weathers.

(7) Duration of normal weather is much higher than that of stormy weather.

(8) Repair is possible in normal weather only.

The assumptions stated above, though quite large in number, are valid for a practical power system. It will be shown in following sections that some more assumptions are made in various approaches for prediction of power system reliability for further simplification of the problem.

CHAPTER - III

EXISTING APPROACHES

3.1 Simplified Approach to Power System Reliability[1]

The method discussed in this section is a block diagram reduction technique where series and parallel subunits are reduced into approximate equivalents assuming an independent process and no repair in the stormy weather. Because of these assumptions the evaluation of reliability becomes very simple.

(a) Series System: Consider a system composed of n dissimilar components connected in series. The equivalent failure rate for ith component is given by

$$D_{avi} = \frac{N}{N+S} D_i + \frac{S}{N+S} D_i' \qquad (1)$$

The symbols used in the above expression are defined in the nomenclature. This is an approximation when $D_i N$ and $D_i' S$ are very small compared to unity as is usually the case in utility practice. The reliability function of the ith component is given by

$$R_i(t) = e^{-D_{avi}\, t}$$

The system will operate only when all components will operate individually, therefore the reliability of the whole system will be

$$R_S(t) = R_1(t).R_2(t) \ \ldots \ R_n(t)$$
$$= e^{-(D_{av1} + D_{av2} + \ldots D_{avn})t}$$

The above expression gives the equivalent rate of failure of the series system as

$$D_S = \sum_{i=1}^{n} D_{avi} \qquad (2)$$

The average or equivalent repair rate U_S can be obtained as follows:

$$\text{Average repair time} = \frac{1}{U_S} = \frac{\text{average anual down time}}{\text{average no. of failures per year}}$$

or

$$U_S = \frac{D_{av1} + D_{av2} + \ldots D_{avn}}{\frac{D_{av1}}{U_1} + \frac{D_{av2}}{U_2} + \ldots \frac{D_{avn}}{U_n}}$$

$$= \frac{D_S}{\sum_{i=1}^{n} D_{avi} \cdot \frac{1}{U_i}} \qquad (3)$$

(b) <u>Parallel System</u>: The equations to be given for parallel systems are limited to two components in parallel. If there are more components connected in parallel they may be treated two at a time, i.e., two of the three parallel components are combined then the equivalent combined with the third and so on. There are four ways in which the parallel system containing two components can fail.

(1) Initial failure is during normal weather and second failure is during normal weather: Since repair times are very short compared with times between storms such that, at most, one weather change is likely during a repair time, system failure rate can be derived as given in the next page.

Failure rate = (long run fraction of time that weather is normal) [(normal weather failure rate of component 1) (probability that a storm does not occur during repair of component 1)(probability that component-2 fails during repair of component 1) + (normal weather failure rate of component 2) (probability that a storm does not occur during repair of component 2) (probability that component 1 fails during repair of component 2)]

$$= \frac{N}{N+S}\left\{D_1\left(1 - \frac{1}{U_1N}\right)\left(\frac{D_2}{U_1}\right) + D_2\left(1 - \frac{1}{U_2N}\right)\left(\frac{D_1}{U_2}\right)\right\}$$

Since NU_1 and NU_2 are much larger than 1, it can be shown that

$$\text{Failure rate} = \frac{N}{N+S} D_1D_2\left(\frac{1}{U_1} + \frac{1}{U_2}\right) \qquad (4)$$

(2) Initial failure is during normal weather and second failure is during stormy weather: System failure rate as a result of first component failure during normal weather and second component failure during stormy weather is

Failure rate = (long run fraction of time that weather is normal) [(normal weather failure rate of 1) (probability that a storm occurs during repair of 1) (probability that 2 fails during that storm) + (term giving other possibility)]

$$= \frac{N}{N+S}\left\{D_1\left(\frac{1}{NU_1}\right)(D_2'S) + D_2\left(\frac{1}{NU_2}\right)(D_1'S)\right\}$$

$$= \frac{N}{N+S}\left(\frac{S}{N}\right)\left(\frac{D_2'D_1}{U_1} + \frac{D_1'D_2}{U_2}\right) \qquad (5)$$

(3) Initial failure is during stormy weather and second failure is during stormy weather: Assuming that storm durations are short compared with the repair times, system failure rate as a result of component failures during the same storm is

Failure rate = (expected number of storms per year)[(probability that 1 fails during a storm) (probability that 2 fails during the same storm) + (term giving other possibility)]

$$= \frac{1}{N + S}\left[(SD_1')(SD_2') + (SD_2')(SD_1')\right]$$

$$= \frac{N}{N + S}\left(\frac{2S^2}{N} D_1'D_2'\right) \qquad (6)$$

It can be noted that probability of a second component failure during a storm is same as the probability of the first failure because of the memoryless nature of the exponential distribution describing storm durations.

(4) Initial failure is during stormy weather and second failure is during normal weather: System failure rate for this case is

Failure rate = (expected no. of storms per year) [(probability 1 fails during a storm) (probability 2 does not fail during storm) (probability 2 fails during repair of 1) + (term giving other possibility)]

$$= \frac{1}{N + S}\left[(SD_1')(1-SD_2')\left(\frac{D_2}{U_1}\right) + (SD_2')(1-SD_1')\left(\frac{D_1}{U_2} \right)\right]$$

Since $1 \gg SD_1'$

$$\text{Failure rate} = \frac{N}{N + S}\left(\frac{S}{N} \right)\left(\frac{D_1'D_2}{U_1} + \frac{D_2'D_1}{U_2} \right) \qquad (7)$$

The equivalent failure rate for the parallel system in normal weather is the summation of the possibilities (1) and (4) and, therefore, is given by

$$D_P = D_1D_2\left(\frac{1}{U_1} + \frac{1}{U_2} \right) + \frac{S}{N}\left(\frac{D_1'D_2}{U_1} + \frac{D_2'D_1}{U_2} \right) \qquad (8)$$

Similarly the equivalent system failure rate in the stormy weather is the summation of the possibilities (2) and (3) and is given by

$$D_P' = \frac{D_1D_2'}{U_1} + \frac{D_2D_1'}{U_2} + 2SD_1'D_2' \qquad (9)$$

The equivalent repair rate for the system is given by

$$U_P = U_1 + U_2 \qquad (10)$$

The equivalents for single state weather can be achieved by substituting $D_1' = D_2' = S = 0$ in the above expressions.

The above approach assumes independence of system states. This assumption, though makes the process very simple, is not valid for a practical system because the occurance of one event depends upon the occurance of other or in other words the process is dependent. Billinton et.al[3] applied the concept of Markov process, which considers the dependence of events, for calculation of power system reliability. The method is described in the next section.

3.2 Application of Markov Process to Power System Reliability

A process is said to be independent of the knowledge of the outcome of any preceding experiment does not affect the prediction for the next experiment. For a Markov process the events are dependent and the knowledge of the immediate past influences the prediction of following event. The theoretical development of the Markov approach is discussed in considerable detail by Feller[4] and by Sandler[5] particularly with regard to processes that are discrete in space and continuous in time. This method also considers a system which is continuous in time but discrete in space as is usually the case in power systems. The Markov technique can be applied to a single unit system as follows:

The state space diagram for a single unit case is shown in Fig.1. The symbols used are defined in the nomenclature.

Considering exponential failure and repair distributions and negligible incremental time interval dt,

$$P_0(t + dt) = P_0(t)(1-D\,dt)(1-n\,dt) + P_1(t)m\,dt(1-D'\,dt) + P_2(t)U\,dt(1-n\,dt)$$

$$P_1(t + dt) = P_0(t)n\,dt(1-D\,dt) + P_1(t)(1-m\,dt)(1-D'\,dt) + P_3(t)\,U'dt(1-m\,dt)$$

$$P_2(t + dt) = P_0(t)D\,dt(1-n\,dt) + P_2(t)(1-U\,dt)(1-n\,dt) + P_3(t)m\,dt(1-U'\,dt)$$

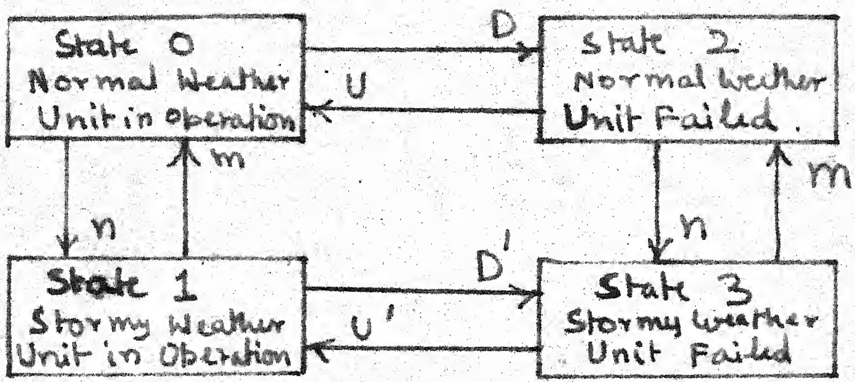

Fig. 1 : State Space Diagram
For One Unit Case

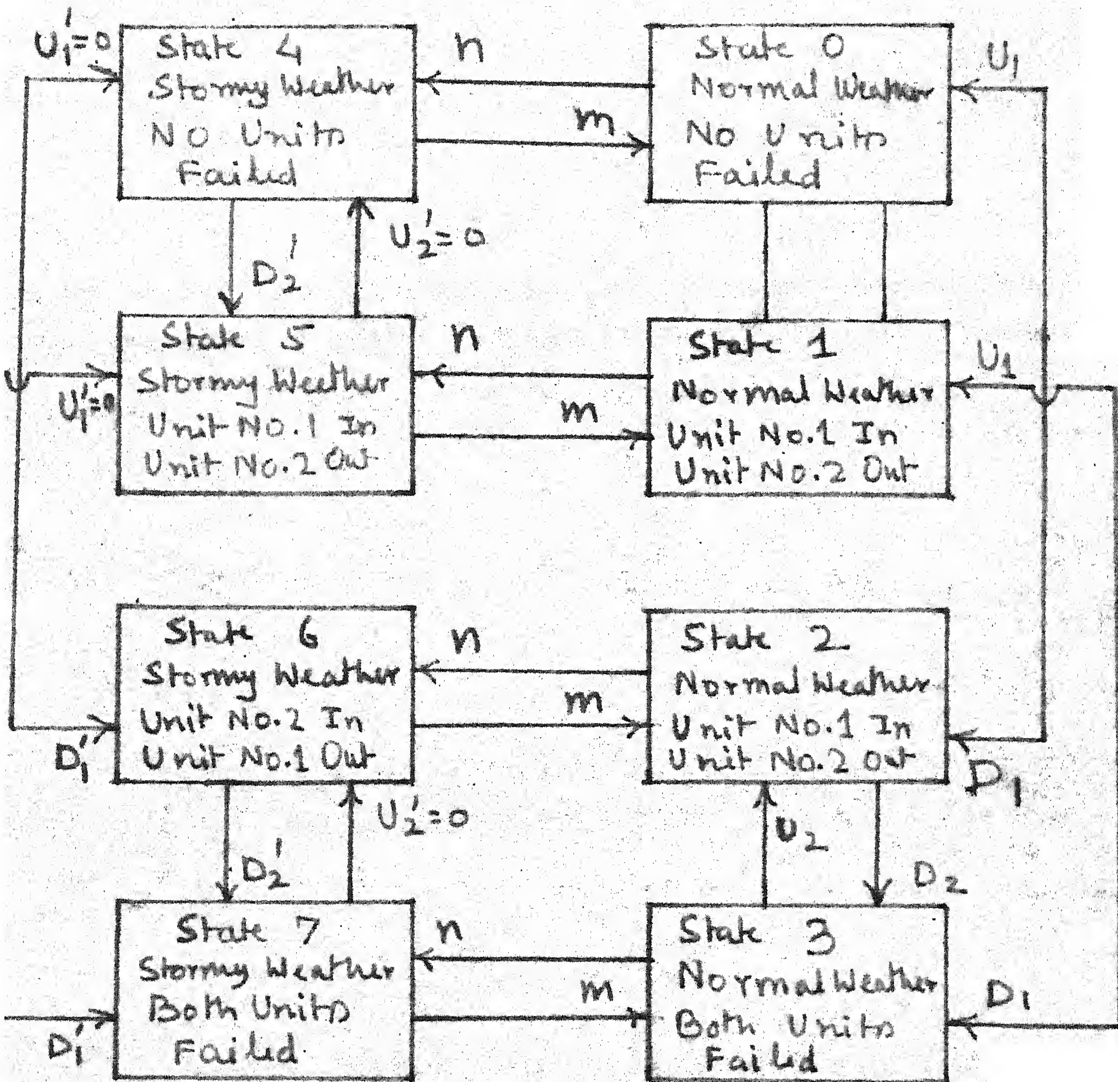

Fig. 2: State Space Digram For
Two Unit Case

$$P_3(t + dt) = P_1(t)\ D'\ dt(1-m\ dt) + P_2(t)\ n\ dt(1-U\ dt)$$
$$+ P_3(t)(1-m\ dt)(1-U'\ dt)$$

Neglecting higher order terms of dt and dividing by dt the above expressions reduce to

$$\frac{P_0(t + dt) - P_0(t)}{dt} = -(D + n)P_0(t) + mP_1(t) + UP_2(t)$$

$$\frac{P_1(t + dt) - P_1(t)}{dt} = nP_0(t) - (m + D')P_1(t) + U'P_3(t)$$

$$\frac{P_2(t + dt) - P_2(t)}{dt} = DP_0(t) - (U + n)\ P_2(t) + mP_3(t)$$

$$\frac{P_3(t + dt) - P_3(t)}{dt} = D'P_1(t) + nP_2(t) - (U' + m)P_3(t)$$

Taking the limit as dt tending to zero in the above equations the differential equation in the matrix form can be written as

$$\begin{bmatrix} P_0'(t) \\ P_1'(t) \\ P_2'(t) \\ P_3'(t) \end{bmatrix} = \begin{bmatrix} -(D+n) & m & U & 0 \\ n & -(m+D') & 0 & U' \\ D & 0 & -(U+n) & m \\ 0 & D' & n & -(U'+m) \end{bmatrix} \times \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix} \qquad (11)$$

The stochastic transitional probability matrix will be

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1-D-n & n & D & 0 \\ m & 1-m-D & 0 & D' \\ U & 0 & 1-U-n & n \\ 0 & U' & m & 1-U'-m \end{bmatrix} \end{array} \qquad (12)$$

To obtain the mean time to failure it is not important which state the system failed in, only that it did fail. Both states 2 and 3 can be considered as absorbing states which are states for system failure. The truncated matrix X is obtained by deleting states 2 and 3 from the matrix P and is given by

$$X = \begin{array}{c c} & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 0 \\ 1 \end{array} & \begin{bmatrix} 1-D-n & n \\ m & 1-m-D' \end{bmatrix} \end{array} \qquad (13)$$

If I is the identity matrix then

$$[I - X]^{-1} = \frac{1}{DD'+Dm+Dn} \begin{array}{c c} \begin{array}{cc} 0 & 1 \end{array} & \\ \begin{bmatrix} m+d' & n \\ m & D+n \end{bmatrix} & \begin{array}{c} 0 \\ 1 \end{array} \end{array} \qquad (14)$$

Billinton has shown that starting from state 0 the equivalent rate of failure is

$$D_{av} = \frac{DD' + Dm + D'n}{m + D' + n} \qquad (15)$$

and starting from state 1 the equivalent rate of failure is

$$D'_{av} = \frac{DD' + Dm + D'n}{m + D + n} \qquad (16)$$

The process of obtaining equivalent failure rate for a multiple unit system is exactly similar. The differential equation for such a system will be of the form

$$P'(t) = T\,P(t) \qquad (17)$$

In the equation (17) P(t) is the transient probability vector and T is defined as the system matrix of the system. The order of the above system matrix for a n unit system is $(2^{n+1}, 2^{n+1})$ which becomes very large as n increases and hence it becomes too cumbersome to obtain its inverse in order to get the mean time to failure of the system. Therefore this method, though accurate, is very difficult when applied to a system having more number of units.

The results obtained by the method described in this section and by simplified method of Patton are calculated on a digital computer for typical series and parallel configurations and given in Table 1. Because of simplifying assumptions made Patton's method gave widely differing equivalent rate of failures for the different cases considered. This clearly shows that the assumption of independence is not valid in practical power system In the next chapter an accurate method for obtaining the series and parallel equivalents based on transitional probability matrix obtained from Markov technique, is described. The procedure combines the merits of Patton's and Billinton's methods and can b employed both for a simple and complex configurations with equal ease.

TABLE 1 D_{eq} for Multiple Unit System with D_{av} = 0.5 fail./yr., S = 1.5 hrs., N = 200 hrs.

% of Stormy Weather Failure	Duration of Repair (in hrs)	Deq For Two Units in Parallel (Fail / Yr): Existing Approximate Method	Deq For Two Units in Parallel (Fail / Yr): Block Reduction Method Using Markov Process	Deq For Two Units in Parallel (Fail / Yr): Accurate Method Using Markov Process	Deq For Three Units in Parallel (Fail. / Yr): Existing Approximate Method	Deq For Three Units in Parallel (Fail. / Yr): Block Reduction Method Using Markov Process	Deq For Three Units in Parallel (Fail. / Yr): Accurate Method Using Markov Process	Deq For Two Units in Para[llel] with Third in Series (Fail/[Yr]): Existing Approximate Method	Deq For Two Units in Para[llel] with Third in Series (Fail/[Yr]): Block Reduction Method Using Markov Process	Deq For Two Units in Para[llel] with Third in Series (Fail/[Yr]): Accurate Method Using Markov
0.0	2.5	1.437×10^{-4}	1.437×10^{-4}	1.437×10^{-4}	3.100×10^{-8}	3.100×10^{-8}	3.100×10^{-8}	0.50014	0.50014	0.5001
	5.0	2.875×10^{-4}	2.873×10^{-4}	2.673×10^{-4}	1.240×10^{-7}	1.240×10^{-7}	1.240×10^{-7}	0.50028	0.50028	0.5002
20	2.5	5.865×10^{-4}	6.002×10^{-4}	6.002×10^{-4}	3.455×10^{-6}	2.354×10^{-6}	2.274×10^{-6}	0.50058	0.50037	0.5003
	5.0	7.131×10^{-4}	7.434×10^{-4}	7.434×10^{-4}	3.840×10^{-6}	2.675×10^{-6}	2.274×10^{-6}	0.50071	0.50052	0.5005
40	2.5	1.943×10^{-3}	1.957×10^{-3}	1.957×10^{-3}	2.589×10^{-5}	1.751×10^{-5}	1.734×10^{-5}	0.50194	0.50104	0.5010
	5.0	2.347×10^{-3}	2.099×10^{-3}	2.099×10^{-3}	2.713×10^{-5}	1.851×10^{-5}	1.779×10^{-5}	0.50204	0.50118	0.5011
60	2.5	4.215×10^{-3}	4.196×10^{-3}	4.196×10^{-3}	8.484×10^{-5}	5.743×10^{-5}	5.775×10^{-5}	0.50421	0.50211	0.5020
	5.0	4.290×10^{-3}	4.336×10^{-3}	4.336×10^{-3}	8.747×10^{-5}	5.951×10^{-5}	5.838×10^{-5}	0.50429	0.50225	0.5022
80	2.5	7.401×10^{-3}	7.300×10^{-3}	7.300×10^{-3}	1.968×10^{-4}	1.335×10^{-4}	1.359×10^{-4}	0.50740	0.50355	0.5035
	5.0	7.441×10^{-3}	7.437×10^{-3}	7.437×10^{-3}	2.013×10^{-4}	1.370×10^{-4}	1.365×10^{-4}	0.50740	0.50370	0.5036
100	2.5	1.150×10^{-2}	1.125×10^{-2}	1.125×10^{-2}	3.775×10^{-4}	2.566×10^{-4}	2.645×10^{-4}	0.51150	0.50535	0.5052
	5.0	1.150×10^{-2}	1.138×10^{-2}	1.138×10^{-2}	3.843×10^{-4}	2.621×10^{-4}	2.645×10^{-4}	0.51150	0.50548	0.5053

CHAPTER - IV

BLOCK DIAGRAM REDUCTION TECHNIQUE USING MARKOV PROCESS

4.1 Introduction

The preceding chapter gave methods which were either simple but inaccurate or too difficult though accurate. A compromise between these two features, i.e., simplification and accuracy, is the main objective of this method. The procedure is same as the existing simplified approach except for the difference that this method obtains series and parallel equivalents based on Markov process and therefore takes into account the dependence of the states of the system. However, certain degree of independence still prevails because of the assumption that the system can be simplified by taking series, parallel and other equivalents if any, i.e. two or three elements are taken at a time to form an equivalent single unit and then the equivalent combined with others to form another equivalent and thus the whole system in the end reduces to a single unit.

4.2 Series and Parallel Equivalents

Presuming that repair in stormy weather is not possible the state space diagram for two elements is shown in Fig.2. The symbols used are defined in the nomenclature. Proceeding directly from the state space diagram the transitional probability matrix P and system matrix T for two elements are given by the following

two equations:

$$
P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{bmatrix}
1-D_1-D_2-n & D_2 & D_1 & 0 & n & 0 & 0 & 0 \\
U_2 & 1-D_1-U_2-n & 0 & D_1 & 0 & n & 0 & 0 \\
U_1 & 0 & 1-U_1-D_2-n & D_2 & 0 & 0 & n & 0 \\
0 & U_1 & U_2 & 1-U_1-U_2-n & 0 & 0 & 0 & n \\
m & 0 & 0 & 0 & 1-D_1'-D_2'-m & D_2' & D_1' & 0 \\
0 & m & 0 & 0 & 0 & 1-D_1'-m & 0 & D_1' \\
0 & 0 & m & 0 & 0 & 0 & 1-D_2'-m & D_2' \\
0 & 0 & 0 & m & 0 & 0 & 0 & 1-m
\end{bmatrix}
\end{array}
$$

.... (18)

The system matrix T comes out to be the transpose of the matrix (P-I) and therefore,

$$
T = \begin{bmatrix}
-D_1-D_2-n & U_2 & U_1 & 0 & m & 0 & 0 & 0 \\
D_2 & -D_1-U_2-n & 0 & U_1 & 0 & m & 0 & 0 \\
D_1 & 0 & -U_1-D_2-n & U_2 & 0 & 0 & m & 0 \\
0 & D_1 & D_2 & -U_1-U_2-n & 0 & 0 & 0 & m \\
n & 0 & 0 & 0 & -D_1'-D_2'-m & D_2' & D_1' & 0 \\
0 & n & 0 & 0 & 0 & -D_1'-m & 0 & D_1' \\
0 & 0 & n & 0 & 0 & 0 & -D_2'-m & D_2' \\
0 & 0 & 0 & n & 0 & 0 & 0 & -m
\end{bmatrix}
$$

.... (19)

While deriving the equivalent single unit for two units it is always assumed that

two unit system $\equiv$ one unit system (20)

The identity equation in the previous page means that all the independent parameters of a one unit system should be equal to all the similar parameters of the two unit system. The independent parameters in the one unit system are

(1) D_{av} : The rate of failure of the unit starting from a state when unit is working in normal weather.

(2) D'_{av} : The rate of failure of the unit starting from a state when unit is working in stormy weather.

(3) U : The repair rate of the unit.

Therefore if D_{avT} is the average rate of failure for two unit system starting from a system when both the components are working in normal weather and D'_{avT} is the average rate of failure for the system starting from a state when both the components are working in stormy weather then,

$$D_{avT} = D_{av} \tag{21}$$

$$D'_{avT} = D'_{av} \tag{22}$$

Substituting the values of D_{av} and D'_{av} from equations (15) and (16) into the above two equations

$$D_{avT} = \frac{DD' + Dm + D'n}{m + D' + n}$$

and $$D'_{avT} = \frac{DD' + Dm + D'n}{m + D + n}$$

Neglecting DD' in comparison to (Dm + D'n) the above two equations reduce to

$$D_{avT} = \frac{Dm + D'n}{m + D'+n} \tag{23}$$

$$D'_{avT} = \frac{Dm + D'n}{m + D + n} \tag{24}$$

Solving equations (23) and (24) for D and D'

$$D = \frac{(m+n)(n-D_{avT})D'_{avT} - nD_{avT}(m+n)}{(m-D'_{avT})(n-D_{avT}) - mn} \tag{25}$$

and

$$D' = \frac{(m-D'_{avT})(m+n)D_{avT} - mD'_{avT}(m+n)}{(m-D'_{avT})(n-D_{avT}) - mn} \tag{26}$$

The above D and D' give the stormy and normal weather failure rates for the equivalent one unit case. In the equations (25) and (26) m and n are known and the only unknowns are D_{avT} and D'_{avT}. The values of D_{avT} and D'_{avT} are derived for a series and parallel system in Appendix 1.1 and 2.1 (given by the equations A1.1, A1.2, A2.5 and A2.7).

After equating the third independent parameter of the single unit system to the equivalent repair rate of the two unit system the average or equivalent one unit repair rate for the parallel and series systems can be derived as shown in Appendix-1.2 and 2.2 (equations A1.11 and A2.11).

While developing the above equivalents it was assumed that repair in stormy weather is zero. This assumption is quite valid in actual practice because in all practical power systems the repair is done only after the storm is over.

Using the above equivalents obtained both for series and parallel configurations the average rate of failures for multiple unit systems for various percentages of stormy weather failures and different repair durations are calculated on digital computer

and given in Table 1. It can be seen from this table that the results obtained by this method are fairly identical to those obtained by Billinton's accurate method. This clearly shows that system reliability can be accurately measured by taking individual series and parallel equivalent blocks. This procedure is quite efficient for a multiunit system.

4.3 Technique for Complex Systems

The method described in the last section can be very well applied to a simple system, but a situation may arise where these simple equivalents can not be directly applied. A very practical example will be ladder type of network as shown in Fig.3(a). In this configuration the elements 1 and 2, 4 and 5 and so on are neither in parallel nor in series. Because of the unability of the above method to reduce such systems a new concept of Star-Delta conversion is developed. Such multiunit systems can be reduced into one equivalent unit by using Star-Delta, series and parallel equivalents. The method is explained by taking the example of the system shown in Fig.3(a). In this system five units are connected in a ladder type of configuration. Nodes a and d are input and output nodes. Since their position is not going to be changed in the reduction process they are called fixed nodes. It can be easily observed that nodes a, b and c are connected in delta and hence they can be converted into an equivalent star as shown in Fig.3(b) Using series equivalents the system of Fig.3(b) can be reduced to

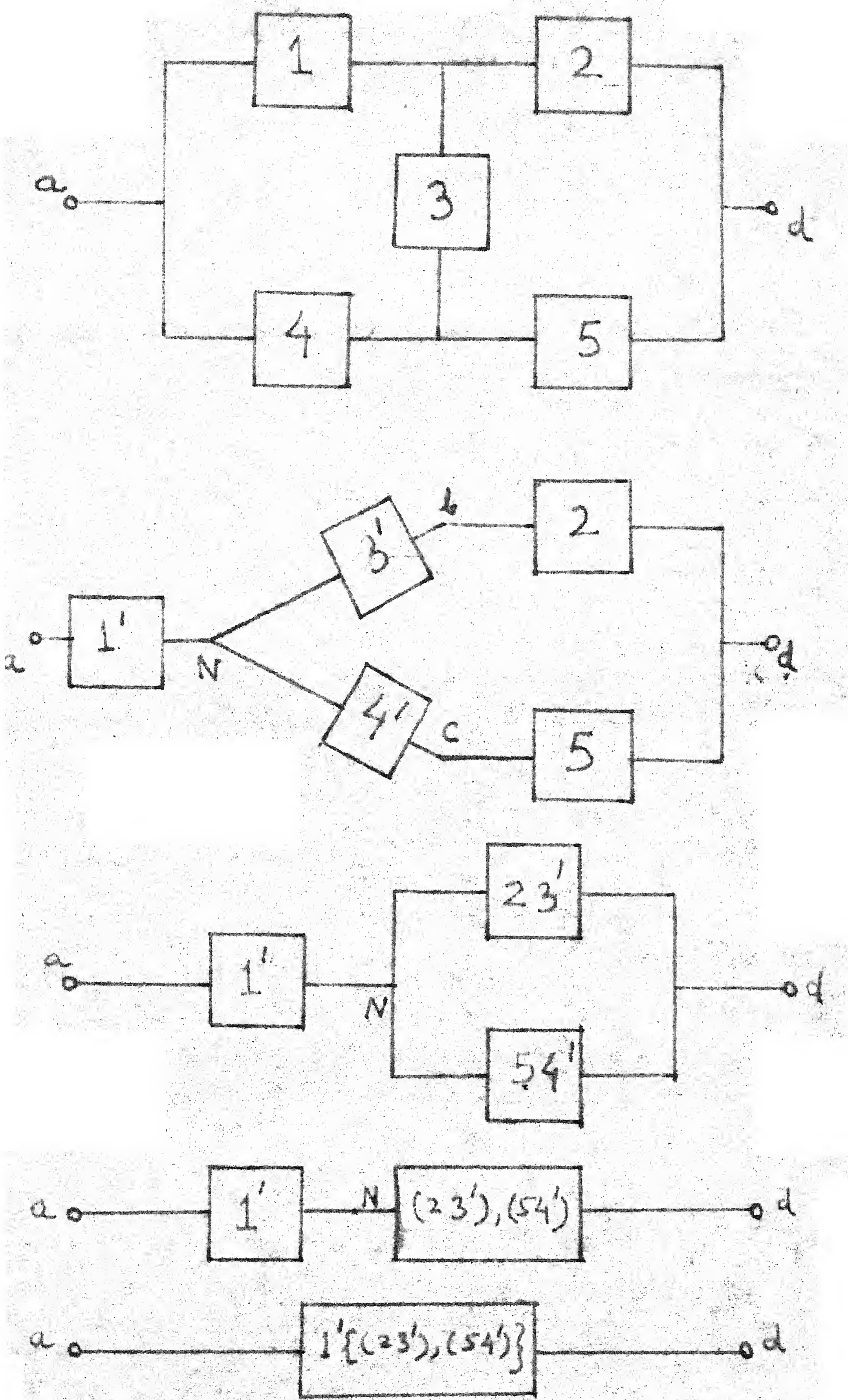

Fig. 3 : Reduction of a Complex System into an equivalent One Unit system.

the system of Fig.3(c). Then elements (2 3') and (5 4') are in parallel and hence can be reduced as shown in Fig.3(d). Finally concept of series equivalents is once again applied to get the equivalent single unit system shown in Fig.3(e). In this way any other large power syst em may be taken and reduced to a small single unit block.

The expressions for star equivalents of a delta model in a single state weather and as well as in two state weather are derived in Appendix 3. The delta equivalents of a star model can be derived using the above expressions. For a two state weather model the assumption of independence of events has been taken into account. Otherwise the problem becomes quite complex and it is practically impossible to obtain analytical expression for star equivalent of the delta model. This concept of Star-Delta conversion, although quite old for electrical networks, is successfully applied to a reliability model for the first time. The success of this approach can be observed in Table 3 where the average failure rates for the complex system of Fig.3(a) are calculated using the above technique and Billinton's technique and then compared. The results obtained by the above two methods come out to be almost identical.

TABLE 2 Single State Weather: Comparison of the accurate method and block diagram reduction method using star-delta conversion (rate of repair of a single unit = 1000 repairs/year).

Rate of failure of a single unit (failures/year)	Equivalent rate of failure for the complex system <u>Accurate Method</u> (Failures/year)	Equivalent rate of failure for the complex system <u>Block Diagram Reduction Method</u> (Fail./yr.)
0.1	3.995×10^{-5}	4.00×10^{-5}
0.2	1.699×10^{-4}	1.60×10^{-4}
0.3	3.698×10^{-4}	3.69×10^{-4}
0.4	6.399×10^{-4}	6.40×10^{-4}
0.5	0.9998×10^{-3}	1.00×10^{-3}

TABLE 3 Two State Weather: Comparison of the accurate method and block diagram reduction method using star-delta conversion (S=1.5 hrs., N=200 hrs., U=1000 rep./yr., D_{av}=0.5 fail./yr.).

Percentage of stormy weather failure	Equivalent rate of failure for the complex system(fail./yr.)	
	Accurate Method	Block Diagram Reduction Method
0.0	1.0×10^{-3}	1.0×10^{-3}
20	1.92×10^{-3}	1.88×10^{-3}
40	4.68×10^{-3}	4.52×10^{-3}
60	9.26×10^{-3}	8.85×10^{-3}
80	1.56×10^{-2}	1.51×10^{-2}
100	2.38×10^{-2}	2.24×10^{-2}

CHAPTER - V

COMPUTER APPLICATION TO THE RELIABILITY ANALYSIS

5.1 Introduction

The techniques discussed in Chapters III and IV for simple systems and for complex systems can be solved on a digital computer. In this chapter a brief review of the existing methods is discussed for the evaluation of reliability measures on computer. A modified flow chart is given for the method described in Chapter IV which can be used to evaluate reliability measure of even complex multiple unit systems.

5.2 Existing Methods

(a) Block Diagram Reduction Technique using Only Series and Parallel Equivalents: For a simple system where series and parallel subunits are well defined the application of computer is fairly easy because the problem is only to indicate the series and parallel blocks. But in case of complex systems where these blocks are not well observable the problem is quite difficult. For such systems Esser et.al[6] have used the concept of tie set and cut set to reduce a complex system into a simple system. This method involves following steps:

(1) Formation of minimal tie sets of the complex systems.

(2) Finding the dual of the above set by interchanging the logical symbols . and + in order to give the minimal cut set for the system.

(3) Representing the system into series and parallel blocks alone with the help of the above dual set.

The method can be made clear by taking the example of the complex system shown in Fig.3(a). In the complex system represent subunit 1 by A, subunit 2 by B, subunit 3 by C, subunit 4 by D and subunit 5 by E. During the first step tie sets are determined by formation of the boolean expression

$$T = A.B + A.C.E + D.E + D.C.B \tag{27}$$

The dual of the above set will be

$$T' = (A + B).(A + C + E).(D + E).(D+C+B)$$

which simplifies to

$$A.D + A.C.E + B.C.D + B.E = \text{Cut Sets} \tag{28}$$

The above expression has a configuration as shown in Fig.4 and does not have a block other than a series or parallel block. Thus the complex system finally reduces to a simple system and hence can be very well solved on the computer using series and parallel equivalents only.

(b) <u>Reliability Evaluation Using Markov Process</u>: The computer program can be made exactly on the lines of the method discussed in section 3.2. The systematic determination of the absorbing or failed states can be done by formation of the cut sets of the system. A detailed discussion of this method is given in Reference 7.

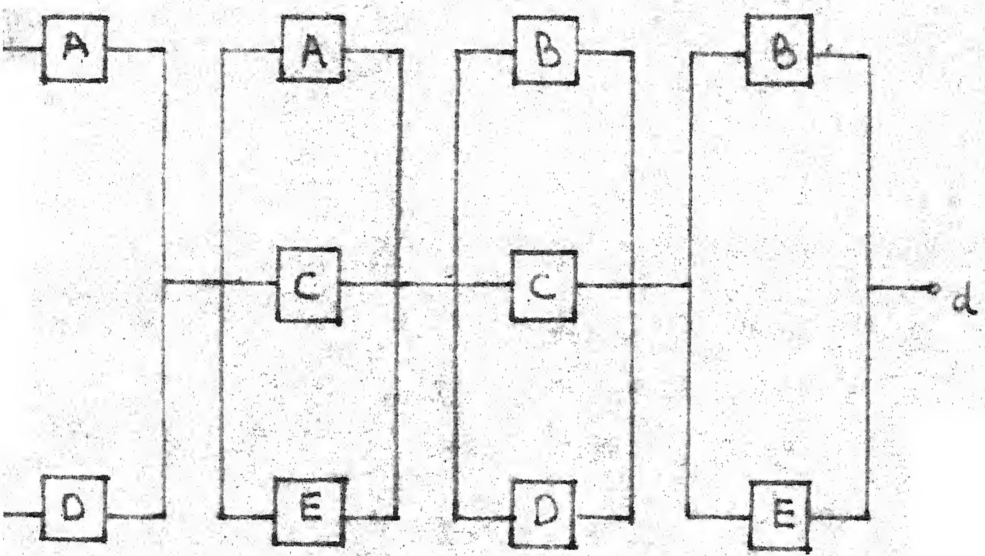

Fig. 4 : Series - Parallel Equivalent
Of The Complex System

5.3 Computer Program Using Series, Parallel and Star-Delta Equivalents

This is also a block diagram reduction method whose objective is to develop a procedure which can form the basis of efficient digital computer programs. The computer flow chart of this method is shown in Fig.5. Input data to the computer program is the topology of the system. By known topology one means the system configuration in which all the nodes are well definedd and the relisbility parameters defined in section 4.2 of the components are given in the form of matrices. In other words one matrix giving all the failure rates in stormy weather, one matrix giving all the failure rates in normal weather and one matrix giving all the repair rates of the components connected between system nodes should be given as the input data. The result of this computer program will also be in the form of three matrices which will have all the elements zero except those between the input and output nodes. These will give equivalent stormy weather failure, normal weather failure and repair rate of the system. This computer program, though more time consuming than the existing method using tie sets, is more advantageous in respect that it proceeds directly from the topology of the system and hence saves the manual labour in formation of minimal tie and cut sets. However, it is not more accurate because the Star-Delta conversion used in this program involves certain degree of approximation as discussed in section 4.2.

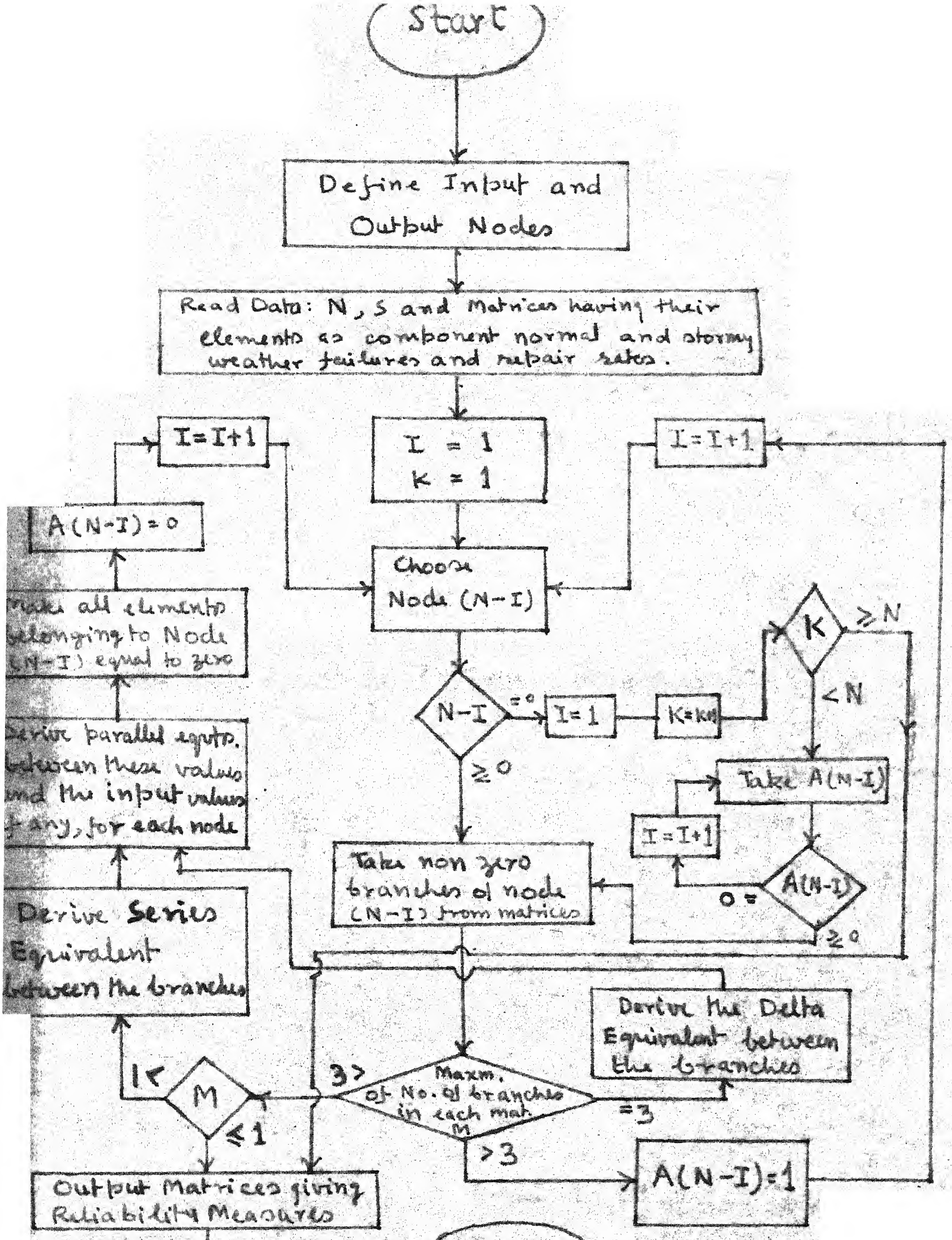

Fig. 5 : A Computer Flow Diagram For Evaluation of Reliability Measures

CHAPTER - VI

CONCLUSION

An accurate method for the evaluation of power system relibability must consider the dependency of the transitional probabilities. The existing simple approach considers the system to contain independent states hence is considered to be inaccurate. The existing accurate method is considerably difficult because the system matrix to be used for evaluation of power system reliability is very large and it is very much time consuming to find its inverse. The development of equivalent units based on the Markov Process acts as a bridge between accuracy and simplicity and thus simplifies the accurate method by introducing some additional approximation. These approximations simplify the reliability evaluation to a great extent and at the same time do not introduce much error in the results. The concept of Star-Delta conversion used here is quite new to the field of reliability studies. It is shown to be very useful in the reduction of complex systems. In the end computer application has been discussed and a new computer program for the evaluation of reliability is developed. The computer program is quite simple and uses only the topology of the system as its input data.

The problem of systems planning may be expected to become more and more important in the future and to require the development of more advanced techniques. A future work in this area can be to predict the number of minimum standby units required in a

power plant keeping the cost to be one of the important factors in the overall optimization.

NOMENCLATURE

Single Unit Case

D, U	normal weather failure and repair rates.
D', U'	stormy weather failure and repair rates.
$S = \frac{1}{m}$	expected duration of stormy weather.
$N = \frac{1}{n}$	expected duration of normal weather
$P_r(t)$	probability of system being in rth state at time t (r can vary from 0 to 3).

More than One Unit Case

D_1, D_2, D_A, D_B, D_C, D_{AB}, D_{BC}, D_{AC}	normal weather failure rates of components- 1, 2, A, B, C, AB, BC and AC.
D'_1, D'_2, D'_A, D'_B, D'_C, D'_{AB}, D'_{BC}, D'_{AC}	stormy weather failure rates of components- 1, 2, A, B, C, AB, BC and AC.
U_1, U_2, U_A, U_B, U_C, U_{AB}, U_{BC}, U_{AC}	normal weather repair rates for components- 1, 2, A, B, C, AB, BC and AC.
D_{eq}	Equivalent rate of failure.

APPENDIX - I

EQUIVALENT MODEL FOR TWO SERIES ELEMENTS

1.1 2-State Weather

(a) The MTTF of the system is obtained by truncating the stochastic transitional probability matrix for two units by elemanating the absorbing states 1, 2, 3, 5, 6 and 7.

In Reference 3 it has been derived that MTTF for two elements in series starting from state 0 is given by

$$M_{S0} = \frac{D_1'+D_2'+m+n}{(D_1+D_2)(D_1'+D_2'+m)+n(D_1'+D_2')}$$

Therefore, average rate of failure, which is inverse of MTTF, for a series model starting from state '0' is given by

$$D_{avS2} = \frac{(D_1+D_2)(D_1'+D_2'+m)+n(D_1'+D_2')}{(D_1'+D_2'+m+n)} \qquad \text{(A1.1)}$$

Similarly average rate of failure starting from state '4' is given by

$$D'_{avS2} = \frac{(D_1+D_2)(D_1'+D_2'+m)+n(D_1'+D_2')}{(D_1+D_2+m+n)} \qquad \text{(A1.2)}$$

(b) Equivalent repair rate assuming repair is possible in normal weather only can be derived as following:

The steady state availability in rth state is given by[7]

$$q_r = \frac{|Y_{1r}|}{|Y|} \qquad \text{(A1.3)}$$

In the expression (A1.3) $|Y|$ is the determinant of the system matrix in which the first row is replaced by 1,1,...1 and $|Y_{ir}|$ is the determinant of matrix Y in which the (1+r)th column is replaced by 1,0,0,...0.

For series case the system is available in states 0 and 4. The availability in the state 0 is

$$q_0 = \frac{|Y_{10}|}{|Y|} \tag{A1.4}$$

where

$$|Y| = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ D_2 & -D_1-U_2-n & 0 & U_1 & 0 & m & 0 & 0 \\ D_1 & 0 & -U_1-D_2-n & U_2 & 0 & 0 & m & 0 \\ 0 & D_1 & D_2 & -U_1-U_2-n & 0 & 0 & 0 & m \\ n & 0 & 0 & 0 & -D_1'-D_2'-m & 0 & 0 & 0 \\ 0 & n & 0 & 0 & D_2' & -D_1'-m & 0 & 0 \\ 0 & 0 & n & 0 & D_1' & 0 & -D_2'-m & 0 \\ 0 & 0 & 0 & n & 0 & D_1' & D_2' & -m \end{vmatrix}$$

and

$$|Y_{10}| = \begin{vmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & -D_1-U_2-n & 0 & U_1 & 0 & m & 0 & 0 \\ 0 & 0 & -U_1-D_2-n & U_2 & 0 & 0 & m & 0 \\ 0 & D_1 & D_2 & -U_2-U_1-n & 0 & 0 & 0 & m \\ 0 & 0 & 0 & 0 & -D_1'-D_2'-m & 0 & 0 & 0 \\ 0 & n & 0 & 0 & D_2' & -D_1'-m & 0 & 0 \\ 0 & 0 & n & 0 & D_1' & 0 & -D_2'-m & 0 \\ 0 & 0 & 0 & n & 0 & D_1' & D_2' & -m \end{vmatrix}$$

The value of these determinants is found to be

$$Y = -my_1+my_2-(m+n)y_3+m(D_1'+D_2'+m+n)y_4-(D_1'+m)y_5+(D_2'+m)y_6 \qquad (A1.5)$$

where

$$y_1 = (D_1'+m)\Big[\{n(D_1'+D_2')+(D_1+D_2)(D_1'+D_2'+m)\}U_1\{D_2'(D_2+n)+mD_2\}$$
$$+U_1{}^2D_2'\{n(D_1'+D_2')+D_2(D_1'+D_2'+m)\}+U_1{}^2m\{nD_2'+D_2(D_1'+D_2'+m)\}\Big]$$
$$+U_1U_2(D_2'+m)\left[D_2(D_1'+D_2'+m)(D_1'+m)+nD_2'm\right]$$

$$y_2 = -(D_2'+m)\Big[\{n(D_1'+D_2')+(D_1+D_2)(D_1'+D_2'+m)\}U_2\{D_1'(D_1+n)+mD_1\}$$
$$+U_2{}^2D_1'\{n(D_1'+D_2')+D_1(D_1'+D_2'+m)\}+U_2{}^2m\{nD_1'+D_1(D_1'+D_2'+m)\}\Big]$$
$$-U_1U_2(D_1'+m)\left[D_1(D_1'+D_2'+m)(D_2'+m)+nD_1'm\right]$$

$$y_3 = \left[mD_1(U_1+D_2)+D_1D_2'(U_1+D_2+n)\right]\left[(D_1'+m)D_2(D_1'+D_2'+m)+mnD_2'\right]$$
$$+\left[\{nD_1'+D_1(D_1'+D_2'+m)\}\{(D_2+n)D_2'+mD_2\}+U_1D_2'nD_1'\right]\left[(D_1+U_2)(D_1'+m)\right.$$
$$\left.+nD_1'\right]+\left[mD_1'(U_1+D_2)+D_1'D_2'(U_1+D_2+n)\right]\left[nD_2'(D_1+U_2+n)\right.$$
$$\left.+nD_2(D_1'+D_2'+m)\right]$$

$$y_4 = -U_1U_2\left[(D_1'+m)(D_2'+m)(D_1+D_2+U_1+U_2)+nD_2'(D_1'+m)+nD_1'(D_2'+m)\right]$$

$$y_5 = nU_1\left[D_2'(D_2+n)+mD_2\right]\left[nD_1'+D_1(D_1'+m)+D_2(D_1'+D_2'+m)+D_2'(D_1+U_2+n)\right]$$
$$+nU_1{}^2\Big[D_2'\{nD_1'+D_2(D_1'+D_2'+m)+D_2'(U_2+n)\}+m\{D_2(D_1'+D_2'+m)$$
$$+D_2'(U_2+n)\}\Big]+nU_1\left[U_2D_2'+mU_2\right]\left[D_2(D_1'+D_2'+m)+D_2'(D_1+U_2+n)\right]$$

$$y_6 = -nU_2\left[D_1'(D_1+n)+mD_1\right]\left[nD_2'+D_2(D_2'+m)+D_1(D_1'+D_2'+m)+D_1'(D_2+U_1+n)\right]$$
$$-nU_2{}^2\Big[D_1'\{nD_2'+D_1(D_1'+D_2'+m)+D_1'(U_1+n)\}+m\{D_1(D_1'+D_2'+m)$$
$$+D_1'(U_1+n)\}\Big]-nU_2\left[U_1D_1'+mU_1\right]\left[D_1(D_1'+D_2'+m)+D_1'(D_2+U_1+n)\right]$$

and

$$|Y_{10}| = -U_1U_2m(D_1'+D_2'+m)\big[(D_1'+m)(D_2'U_1+D_2D_2'+D_2'n+mU_1+mD_2$$
$$+D_2'U_2+mU_2)+(D_2'+m)(D_1D_1'+mD_1+nD_1')\big]$$

Availability in the state 4 is given by

$$q_4 = \frac{|Y_{14}|}{|Y|} \tag{A1.6}$$

$|Y_{14}|$ is obtained by replacing the 5th column of $[Y]$ by 1,0,0,...0. It comes out to be

$$|Y_{14}| = -U_1U_2mn\big[(U_1+U_2+D_1+D_2)(D_1'+m)(D_2'+m)nD_2'(D_1'+m)$$
$$+nD_1'(D_2'+m)\big] \tag{A1.7}$$

Hence total steady state availability in the series case is

$$q_{S2} = q_0 + q_4 = \frac{|Y_{10}| + |Y_{14}|}{|Y|} \tag{A1.8}$$

The steady state unavailability for series case is given by

$$\bar{q}_{S2} = 1 - q_{S2} \tag{A1.9}$$

For exponentially distributed MTTF and repair time the unavailability of a system is the ratio of equivalent failure rate and equivalent repair rate of the system, therefore,

$$\bar{q}_{S2} = \frac{D_{avS2}}{U_{S2}} \tag{A1.10}$$

where U_{S2} is the equivalent repair rate for the series model.

This gives

$$1-q_{S2} = \frac{D_{avS2}}{U_{S2}}$$

or $$U_{S2} = \frac{D_{avS2}}{1-q_{S2}}$$

i.e. $$U_{S2} = \frac{|Y|\, D_{avS2}}{|Y|-|Y_{10}|-|Y_{14}|} \tag{A1.11}$$

1.2 1-State Weather

1-State weather is equivalent to 2-state weather with $D_1' = D_2' = n = 0$ and $m = 1$.

Equivalent rate of failure is obtained by substituting $D_1' = D_2' = n = 0$ and $m = 1$ in expression (A1.1), i.e., the expression for equivalent failure rate for two-state weather case. Thus the equivalent rate of failure for single state weather model is obtained as

$$D_{avS1} = D_1 + D_2 \tag{A1.12}$$

Resubstituting $D_1' = D_2' = n = 0$ and $m = 1$ in (A1.11) the equivalent repair rate for single state weather series model is given by

$$U_{S1} = \frac{(D_1+D_2)(U_1+D_1)(U_2+D_2)}{D_1(U_2+D_2)+U_1D_2}$$

as U_1, $U_2 >> D_1$, D_2

$$U_{S1} \approx \frac{(D_1+D_2)U_1U_2}{U_2D_1+U_1D_2} \tag{A1.13}$$

The expressions for D_{avS1} and U_{S1} are same as derived by Stanton[7].

APPENDIX - II

EQUIVALENT MODEL FOR TWO UNITS IN PARALLEL

2.1 2-State Weather Considering Repair Rates in Stormy Weather to be Zero

(a) Equivalent Failure Rate: According to Billinton[3] the MTTF is obtained by truncating the stochastic transitional probability matrix by deleting the absorbing states and then inverting the truncated matrix after subtracting identity matrix from it.

Therefore, if the truncated matrix is 'X' with z transition states, then let

$$(I-X) = -A = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1z} \\ \cdots & \cdots & \cdots \\ a_{r1} & a_{r2} \cdots & a_{rz} \\ \cdots & \cdots & \cdots \\ a_{z1} & a_{z2} \cdots & a_{zz} \end{bmatrix}$$

The MTTF starting from rth state is given by

$$M_r = \frac{1}{-A}(K_{r1} + K_{r2} + \cdots + K_{rz}) \qquad (A2.1)$$

where I is the identity matrix and $K_{r1}, K_{r2}, \ldots K_{rz}$ are the cofactors of $a_{r1}, a_{r2}, \ldots a_{rz}$ in the above matrix.

It can be easily shown that

$$K_{r1}+K_{r2}+\ldots+K_{rz} = -\begin{vmatrix} a_{11} & a_{12} & \ldots & a_{1z} & 0 \\ a_{21} & a_{22} & \ldots & a_{2z} & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{r1} & a_{r2} & \ldots & a_{rz} & 1 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{z1} & a_{z2} & \ldots & a_{zz} & 0 \\ 1 & 1 & \ldots & 1 & 0 \end{vmatrix}$$

Define the above determinant by $-|\bar{A}|_r$. This matrix is obtained by adding one row with all 1s and one column with all zeros except in the rth row, where it is 1, in the matrix $-A$.

Hence the average rate of failure starting from rth state is

$$D_{avr} = \frac{1}{M_r} = \frac{|A|}{|\bar{A}|_r} \qquad (A2.2)$$

For the present case the determinant $|A|$ comes out to be

$$\begin{aligned} |A| = {} & \left[nD_1'+D_1(D_1'+m)\right]\left[nD_2'+D_2(D_2'+m)\right]\left[(D_1+D_2)(D_1'+D_2'+m)+n(D_1'+D_2')\right] \\ & +U_1(D_2'+m)\left[\{nD_1'+D_1(D_1'+m)\}\{D_2(D_1'+D_2'+m)+nD_2'\}\right] \\ & +U_2(D_1'+m)\left[nD_2'+D_2(D_2'+m)\right]\left[D_1(D_1'+D_2'+m)+nD_1'\right] \\ & +nD_1'D_2'\left[U_2\{nD_2'+(U_1+D_2)(D_2'+m)\}+U_1\{nD_1'+(D_1+U_2)(D_1'+m)\}\right] \end{aligned}$$

$$\ldots \qquad (A2.3)$$

and

$$|\bar{A}|_0 = \left[(U_1+D_2)(D_2'+m)+nD_2'\right]\left[D_2(D_1'+D_2'+m)(n+D_1'+m)+nD_2'(D_1+U_2+m+n)\right] + \left[(U_2+D_1)(D_1'+m)+nD_1'\right]\left[(D_1'+D_2'+m)\left\{(D_2'+m)(U_1+D_2)+mD_1+n(D_1+D_2)+D_1D_2'\right\}+n(D_1'+D_2')(U_1+D_2+n)+mn(D_1'+U_1+D_2)\right] \quad .... \quad (A2.4)$$

Therefore average rate of failure starting from state 0 is

$$D_{avP2} = \frac{|A|}{|A|_0} \quad (A2.5)$$

Starting from state 4 $|\bar{A}|_4$ can be simplified to be

$$|\bar{A}|_4 = y_1'-y_2'+y_3'-y_4'+y_5'-y'_6 \quad (A2.6)$$

where

$$y_1' = m\left[U_2D_2'\left\{(U_1+D_2)(D_2'+m)+nD_2'\right\}+U_1D_1'\left\{(U_2+D_1)(D_1'+m)+nD_1'\right\} + \left\{(U_1+D_2)(D_2'+m)+nD_2'\right\}\left\{(U_2+D_1)(D_1'+m)+nD_1'\right\}\right]$$

$$y_2' = -m\left[D_2'(D_2+n)\left\{(U_1+D_2)(D_2'+m)+nD_2'\right\}+D_1D_2'\left\{D_2(D_2'+m)+nD_2'\right\} + D_2\left\{(U_1+D_2)(D_2'+m)+U_1D_1'\right\}(D_1'+m)\right]$$

$$y_3' = m\left[D_1'(D_1+n)\left\{(U_2+D_1)(D_1'+m)+nD_1'\right\}+D_2D_1'\left\{D_1(D_1'+m)+nD_1'\right\} + D_1(D_2'+m)\left\{(U_2+D_1)(D_1'+m)+U_2D_2'\right\}\right]$$

$$y_4' = -n\left\{(U_2+D_1)(D_1'+m)+nD_1'\right\}\left\{(U_1+D_2)(D_2'+m)+nD_2'\right\}-D_1\left\{D_2(D_2'+m) + nD_2'\right\}\left\{(U_2+D_1)(D_1'+m)+nD_1'\right\}-D_2\left\{D_1(D_1'+m)+nD_1'\right\}\left\{(U_1 + D_2)(D_2'+m)+nD_2'\right\}$$

$$y_5' = mnD_2\left[U_1D_1'+nD_2'+(U_1+D_2)(D_2'+m)\right]+D_2'\left[\left\{D_2(D_2'+m)+nD_2'\right\}\left\{(D_1+n)(D_1+D_2+U_2+n)\right\}+U_1(D_2'+m)\left\{n(D_1+U_2+n)+D_2(D_1+n)\right\}\right]$$

$$y_6' = -mnD_1\left[U_2D_2'+nD_1'+(U_2+D_1)(D_1'+m)\right]-D_1'\left[\left\{D_1(D_1'+m)+nD_1'\right\}\left\{(D_2+n)(D_1+D_2+U_1+n)\right\}+U_2(D_1'+m)\left\{n(D_2+U_1+n)+D_1(D_2+n)\right\}\right]$$

The average rate of failure starting from state 4 is

$$D'_{avP} = \frac{|A|}{|\bar{A}|_4} \tag{A2.7}$$

(b) <u>Equivalent Repair Rate</u>: The system is unavailable in states 3 and 7. Unavailability in state 3 is given by

$$\bar{q}_3 = \frac{|Y_{13}|}{|Y|} \tag{A2.8}$$

where $|Y|$ has been already derived in Appendix I and $|Y_{13}|$ can be simplified to be

$$|Y_{13}| = -m\left[\left\{(m+D_2')(U_1D_1+D_1D_2)+nD_2'D_1\right\}\left\{(D_1'+m)(D_1'+D_2'+m)D_2+D_2'mn\right\}\right]-m\left[nD_1'D_2'U_1+\left\{nD_1'+D_1(D_1'+D_2'+m)\right\}\left\{nD_2'+D_2(D_2'+m)\right\}\right]\left[(D_1+U_2)(D_1'+m)+nD_1'\right]-m\left[U_1D_1'(m+D_2')+D_1'\left\{nD_2'+D_2(D_2'+m)\right\}\right]\left[nD_2'(D_1+U_2+n)+nD_2(D_1'+D_2'+m)\right]$$

Unavailability in state 7 is given by

$$\bar{q}_7 = \frac{|Y_{17}|}{|Y|} \tag{A2.9}$$

where $|Y_{17}|$ is simplified to be

$$|Y_{17}| = -ny_3-D_1'y_5+D_2'y_6$$

where y_3, y_5 and y_6 are already obtained in Appendix I.

Hence total unavailability for parallel system is given by

$$\bar{q}_P = \bar{q}_3 + \bar{q}_7 = \frac{|Y_{13}| + |Y_{17}|}{|Y|} = \frac{D_{avP2}}{U_{P2}}$$

where U_{P2} is the equivalent repair rate for two state weather model.

From the above expression

$$U_{P2} = \frac{D_{avP2} \quad |Y|}{|Y_{13}| + |Y_{17}|} \tag{A2.10}$$

2.2 <u>Single State Weather</u>

Substituting $D_1' = D_2' = n = 0$ and $m = 1$ in equation (A2.5) the equivalent rate of failure for single state environment parallel model is given by

$$D_{avP1} = \frac{D_1 D_2 (U_1+U_2+D_1+D_2)}{(U_1+D_2)D_2+D_1(U_2+D_1)+(U_2+D_1)(U_1+D_2)}$$

Since $U_1, U_2 \gg D_1, D_2$

$$D_{avP1} \simeq \frac{D_1 D_2 (U_1+U_2)}{U_1 U_2} \tag{A2.11}$$

The equivalent repair rate can be obtained by re-substituting $D_1' = D_2' = n = 0$ and $m = 1$ in equation (A2.9). Henc

$$U_{P1} = \frac{(U_1+U_2)(U_1+D_1)(U_2+D_2)}{U_1 U_2}$$

$$\approx (U_1+U_2) \tag{A2.12}$$

These expressions are same as those derived by Stanton[7].

APPENDIX - III

STAR-DELTA CONVERSION

Consider elements AB, BC and AC connected in delta have an equivalent star with elements A, B and C as shown in Fig. A1. The delta equivalent of the star model for single state as well as for two state weather case is obtained in the following sections:

3.1 Single State Weather

In the Fig.A1

A and C in series $\equiv$ (AB and BC in series)in parallel with AC

Using equations (A1.12), (A1.13), (A2.11) and (A2.12)

$$D_A + D_C = (D_{AB}+D_{BC})D_{AC}\left[\frac{1}{U_{AC}} + \frac{D_{AB}U_{BC}+D_{BC}U_{AB}}{U_{AB}U_{BC}(D_{AB}+D_{BC})}\right]$$

$$= \frac{D_{AC}}{U_{AC}U_{AB}U_{BC}}\left[D_{AB}U_{BC}(U_{AB}+U_{AC})+D_{BC}U_{AB}(U_{BC}+U_{AC})\right] \qquad (A3.1)$$

Similarly by symmetry the expressions for (B and C) and (A and B) in series are

$$D_B + D_C = \frac{D_{BC}}{U_{AC}U_{AB}U_{BC}}\left[D_{AB}(U_{AB}+U_{BC})U_{AC}+D_{AC}U_{AB}(U_{BC}+U_{AB})\right] \qquad (A3.2)$$

$$D_A + D_B = \frac{D_{AB}}{U_{AC}U_{AB}U_{BC}}\left[D_{AC}U_{BC}(U_{AB}+U_{AC})+D_{BC}U_{AC}(U_{AB}+U_{BC})\right] \qquad (A3.3)$$

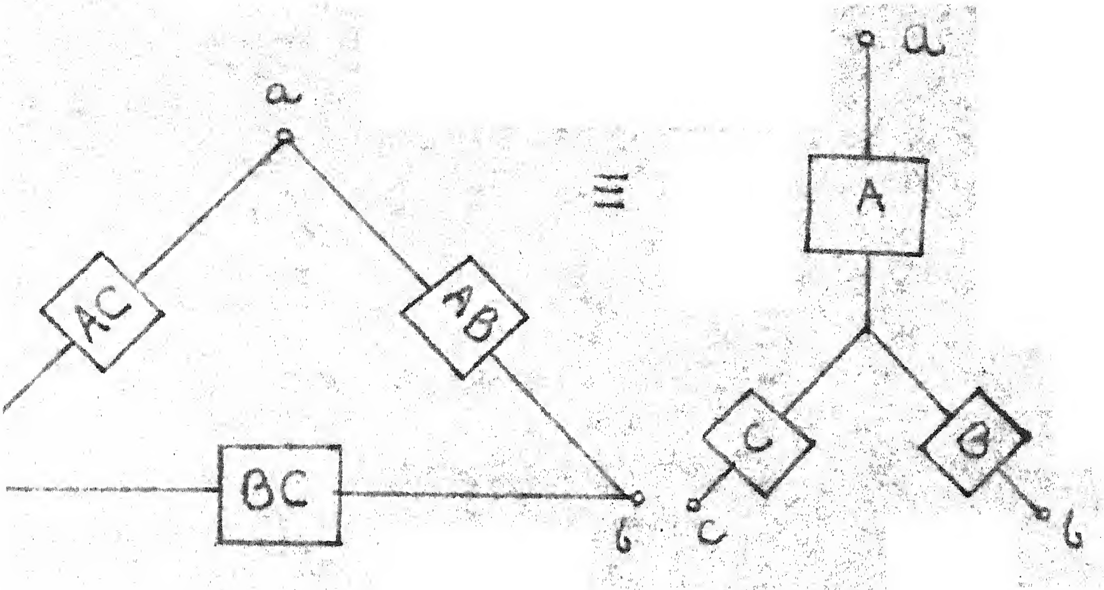

Fig. A1 : Star Equivalent of a Delta Model.

Equations giving equivalent repair rates a

$$\frac{U_A U_C (D_A+D_C)}{(D_A U_C+D_C U_A)} = U_{AC} + \frac{U_{AB} U_{BC} (D_{AB}+D_{BC})}{D_{AB} U_{BC}+D_{BC} U_{AB}}$$

$$\frac{U_C U_B (D_C+D_B)}{(D_C U_B+D_B U_C)} = U_{BC} + \frac{U_{AB} U_{AC} (D_{AB}+D_{AC})}{D_{AB} U_{AC}+D_{AC} U_{AB}}$$

$$\frac{U_A U_B (D_A+D_B)}{(D_A U_B+D_B U_A)} = U_{AB} + \frac{U_{BC} U_{AC} (D_{BC}+D_{AC})}{D_{AC} U_{BC}+D_{BC} U_{AC}}$$

Solving equations (A3.1) to (A3.6) expression equivalent star failure and repair rates are

$$D_A = \left(\frac{D_{AB} D_{AC}}{U_{AB} U_{AC}} \right)(U_{AB}+U_{AC})$$

$$D_B = \left(\frac{D_{AB} D_{BC}}{U_{AB} U_{BC}} \right)(U_{AB}+U_{BC})$$

$$D_C = \left(\frac{D_{AC} D_{BC}}{U_{AC} U_{BC}} \right)(U_{AC}+U_{BC})$$

and

$$U_A = U_{AC} + U_{AB} \qquad \text{(A}$$

$$U_B = U_{BC} + U_{AB} \qquad \text{(A3.}$$

$$U_C = U_{AC} + U_{BC} \qquad \text{(A3.1}$$

3.2 Two State Weather

It is very difficult to get the expressions for star equivalent for a two state weather model using series and parallel equivalents obtained by Markov technique. In order to

simplify the process following approximations are made

(1) $\frac{S}{N}$ is very small (i.e. $1 \gg \frac{S}{N}$)

(2) The process is independent

(3) Repair is possible only in normal weather.

The last assumption has been already used for a two state weather model.

Series equivalent for two elements is given by[1]

$$D_{eS} = D_1 + D_2 \tag{A3.13}$$

$$D'_{eS} = D'_1 + D'_2 \tag{A3.14}$$

and
$$U_{eS} = \frac{N(D_1+D_2) + S(D'_1 + D'_2)}{N(\frac{D_1}{U_1} + \frac{D_2}{U_2}) + S(\frac{D'_1}{U_1} + \frac{D'_2}{U_2})} \tag{A3.15}$$

or

$$\frac{1}{U_{eS}} = \frac{\left[(\frac{D_1}{U_1} + \frac{D_2}{U_2}) + \frac{S}{N}(\frac{D'_1}{U_1} + \frac{D'_2}{U_2})\right]\left[1 + \frac{S}{N}(\frac{D'_1+D'_2}{D_1+D_2})\right]^{-1}}{(D_1 + D_2)}$$

Since $1 \gg \frac{S}{N}$

$$\frac{1}{U_{eS}} \approx \frac{\frac{D_1}{U_1} + \frac{D_2}{U_2}}{(D_1+D_2)} + \frac{S}{N}\frac{1}{(D_1+D_2)^2}(\frac{1}{U_1} - \frac{1}{U_2})(D_2D'_1 - D_1D'_2)$$

The second term is very small, therefore,

$$\frac{1}{U_{eS}} \approx \frac{\frac{D_1}{U_1} + \frac{D_2}{U_2}}{(D_1+D_2)} \tag{A3.16}$$

For two elements in parallel equivalent is[1]

$$D_{eP} = D_1D_2\left(\frac{1}{U_1} + \frac{1}{U_2}\right) + \frac{S}{N}\left(\frac{D_1'D_2}{U_1} + \frac{D_2'D_1}{U_2}\right) \qquad (A3.17)$$

$$D_{eP}' = \frac{D_1D_2'}{U_1} + \frac{D_2D_1'}{U_2} + 2SD_1'D_2' \qquad (A3.18)$$

$$U_{eP} = U_1 + U_2 \qquad (A3.19)$$

Following the same procedure as in Appendix 3.1 the equations for (A and C), (A and B) and (B and C) in series giving the normal weather failure rates can be written as

$$N(D_A+D_C) = \frac{(D_{AB}+D_{BC})}{U_{AC}}(ND_{AC}+SD_{AC}')+D_{AC}\left[N\left(\frac{D_{AB}}{U_{AB}} + \frac{D_{BC}}{U_{BC}}\right) + S\left(\frac{D_{AB}'}{U_{AB}} + \frac{D_{BC}'}{U_{BC}}\right)\right] \qquad (A3.20)$$

$$N(D_A+D_B) = \frac{(D_{AC}+D_{BC})}{U_{AB}}(ND_{AB}+SD_{AB}')+D_{AB}\left[N\left(\frac{D_{AC}}{U_{AC}} + \frac{D_{BC}}{U_{BC}}\right) + S\left(\frac{D_{AC}'}{U_{AC}} + \frac{D_{BC}'}{U_{BC}}\right)\right] \qquad (A3.21)$$

$$N(D_B+D_C) = \frac{(D_{AB}+D_{AC})}{U_{BC}}(ND_{BC}+SD_{BC}')+D_{BC}\left[N\left(\frac{D_{AB}}{U_{AB}} + \frac{D_{AC}}{U_{AC}}\right) + S\left(\frac{D_{AB}'}{U_{AB}} + \frac{D_{AC}'}{U_{AC}}\right)\right] \qquad (A3.22)$$

On solution of equations (A3.20), (A3.21) and (A3.22)

$$D_A = D_{AB}D_{AC}\left(\frac{1}{U_{AB}} + \frac{1}{U_{AC}}\right) + \frac{S}{N}\left(\frac{D_{AC}D'_{AB}}{U_{AB}} + \frac{D_{AB}D'_{AC}}{U_{AC}}\right) \qquad (A3.23)$$

$$D_B = D_{AB}D_{BC}\left(\frac{1}{U_{AB}} + \frac{1}{U_{BC}}\right) + \frac{S}{N}\left(\frac{D_{AB}D'_{BC}}{U_{BC}} + \frac{D_{BC}D'_{AB}}{U_{AB}}\right) \qquad (A3.24)$$

$$D_C = D_{AC}D_{BC}\left(\frac{1}{U_{AC}} + \frac{1}{U_{BC}}\right) + \frac{S}{N}\left(\frac{D_{AC}D'_{BC}}{U_{BC}} + \frac{D_{BC}D'_{AC}}{U_{AC}}\right) \qquad (A3.25)$$

Equations giving stormy weather failures are

$$D'_A + D'_C = \frac{D_{AC}}{U_{AC}}(D'_{AB} + D'_{BC}) + D'_{AC}\left(\frac{D_{AB}}{U_{AB}} + \frac{D_{BC}}{U_{BC}}\right) + 2SD'_{AC}(D'_{BC} + D'_{AB}) \qquad (A3.26)$$

$$D'_A + D'_B = \frac{D_{AB}}{U_{AB}}(D'_{AC} + D'_{BC}) + D'_{AB}\left(\frac{D_{AC}}{U_{AC}} + \frac{D_{BC}}{U_{BC}}\right) + 2SD'_{AB}(D'_{AC} + D'_{BC}) \qquad (A3.27)$$

$$D'_B + D'_C = \frac{D_{BC}}{U_{BC}}(D'_{AB} + D'_{AC}) + D'_{BC}\left(\frac{D_{AB}}{U_{AB}} + \frac{D_{AC}}{U_{AC}}\right) + 2SD'_{BC}(D'_{AB} + D'_{AC}) \qquad (A3.28)$$

Solving equations (A3.26) to (A3.28)

$$D'_A = \frac{D_{AB}D'_{AC}}{U_{AB}} + \frac{D'_{AB}D_{AC}}{U_{AC}} + 2SD'_{AB}D'_{AC} \qquad (A3.29)$$

$$D'_B = \frac{D_{AB}D'_{BC}}{U_{AB}} + \frac{D'_{AB}D_{BC}}{U_{BC}} + 2SD'_{AB}D'_{BC} \qquad (A3.30)$$

$$D'_C = \frac{D_{AC}D'_{BC}}{U_{AC}} + \frac{D'_{AC}D_{BC}}{U_{BC}} + 2SD'_{AC}D'_{BC} \qquad (A3.31)$$

Since single and two state weather equivalent repair rates for series and parallel model come out to be approximately same, the equations giving repair rates for the equivalent

star model for two state weather case will be same as the equations (A3.4), (A3.5) and (A3.6).

The equations (A3.4), (A3.5) and (A3.6) can be solved for $\frac{1}{U_A}$, $\frac{1}{U_B}$ and $\frac{1}{U_C}$ and written in the following matrix form:

$$\begin{bmatrix} \frac{1}{U_A} \\ \frac{1}{U_B} \\ \frac{1}{U_C} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{1}{D_A} & -\frac{1}{D_A} & \frac{1}{D_A} \\ -\frac{1}{D_B} & \frac{1}{D_B} & \frac{1}{D_B} \\ \frac{1}{D_C} & \frac{1}{D_C} & -\frac{1}{D_C} \end{bmatrix} \times$$

$$\begin{bmatrix} \dfrac{(D_{AB}U_{BC}+D_{BC}U_{AB})(D_A+D_C)}{D_{AB}U_{BC}(U_{AB}+U_{AC})+D_{BC}U_{AB}(U_{AC}+U_{BC})} \\[2ex] \dfrac{(D_{AB}U_{AC}+D_{AC}U_{AB})(D_B+D_C)}{D_{AB}U_{AC}(U_{AB}+U_{BC})+D_{AC}U_{AB}(U_{AC}+U_{BC})} \\[2ex] \dfrac{(D_{AC}U_{BC}+D_{BC}U_{AC})(D_A+D_B)}{D_{AC}U_{BC}(U_{AC}+U_{AB})+D_{BC}U_{AC}(U_{AB}+U_{BC})} \end{bmatrix} \quad (A3.32)$$

Substituting the values of D_A, D_B and D_C for equations (A3.23),(A3.24) and (A3.25) and making the approximation that $1 \gg \frac{S}{N}$, it can be shown that

$$U_A = U_{AC} + U_{AB} \quad (A3.33)$$

$$U_B = U_{AB} + U_{BC} \quad (A3.34)$$

$$U_C = U_{AC} + U_{BC} \quad (A3.35)$$

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